

STUDY OF SIMPLE REACTIVE SERIES CIRCUITS USING MATLAB SOFTWARE PACKAGE

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Abstract: This paper presents a modern method for approaching the electrical circuits using the MATLAB-SIMULINK package programs. The simple series circuits which are switched on a DC voltage at the initial moment are presented below. We can determine the current variation forms and the reactive elements voltage, by using this virtual medium. Each presented case contains an analytical presentation of the problem, but it also contains electrical diagrams of electrical parameters. The diagrams were obtained by different methods which use this programs package.

Key words: Simulink, simulation model, differential equation, diagrams, SimPowerSystems.

1. INDUCTIVE SERIES CIRCUITS

We will consider the RL series circuit with concentrated parameters from. At the initial moment, the k circuit switcher is closed and we intend to study the behavior and variation of circuit electrical parameters after connection [4], [7].

The differential equation which corresponds to the transitory regime immediately after closing is the following:

$$Ri + L \frac{di}{dt} = E \quad (1)$$

The current expression through the circuit after closing is the solution of the differential equation of the circuit (1).

$$i(t) = \frac{E}{R} (1 - e^{-\frac{t}{T}}) \quad (2)$$

where:
$$T = \frac{L}{R} \quad (3)$$

and it represents the time constant of the circuit.

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The voltage expression on the coil after connection is:

$$u(t) = L \frac{di}{dt} = E e^{-\frac{t}{T}} \quad (4)$$

1.1. SIMULINK model of the circuit

The SIMULINK model of the circuit after closing was done on grounds of equation (1) where the derivative of the current was separated.

$$\frac{di}{dt} = \frac{1}{L} (E - Ri) \quad (5)$$

The SIMULINK model obtained in this way is shown in Fig.1 and was created in order to allow the drawing of the current diagrams through the circuit and the voltage on the coil, for different values of the R resistance and the E D.C. voltage.

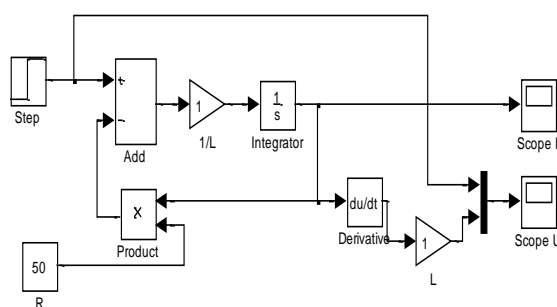


Fig.1. SIMULINK model of the inductive circuit at closing

The k switcher is realized using a voltage step signal which is applied at the terminal of the circuit, to simulate the closing of the k switcher [2].

We will obtain the diagrams from Fig.2 and Fig.3.

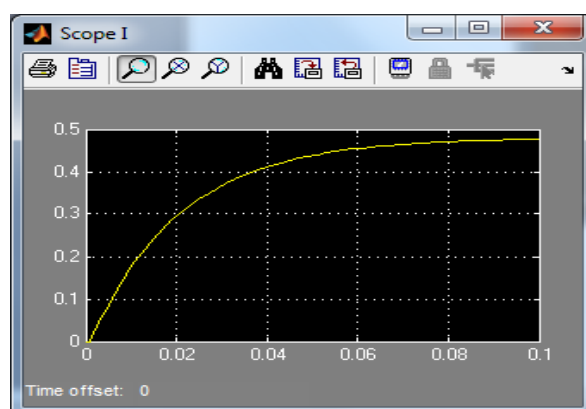


Fig.2. Voltage variation of the coil after closing

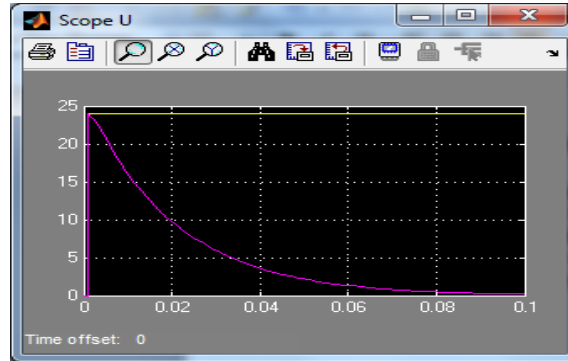


Fig.3. Current variation in the circuit

2. STUDY OF THE CAPACITIVE SERIES CIRCUITS

We will consider the RC series circuit with concentrated parameters. At the initial moment, the circuit switcher is closed and we want to study the current variation through the circuit and the capacitor voltage variation, after connection.

The integral equation which corresponds to the transitory regime immediately after closing is given by:

$$Ri + \frac{1}{C} \int idt = E \quad (6)$$

Equation (6) is equivalent with differential equation:

$$RC \frac{du}{dt} + u = E \quad (7)$$

The expression of the capacitor voltage after closing is the solution of differential equation (7):

$$u(t) = E(1 - e^{-\frac{t}{T}}) \quad (8)$$

where $T = RC$, represents the time constant of the circuit.

The circuit current after closing is given by:

$$i = C \frac{du}{dt} = \frac{E}{R} e^{-\frac{t}{T}} \quad (9)$$

2.1 SIMULINK model of the circuit

The SIMULINK model of the circuit after closing was done on grounds of equation (7) where the voltage derivative was separated.

$$\frac{du}{dt} = \frac{1}{RC}(E - u) \quad (10)$$

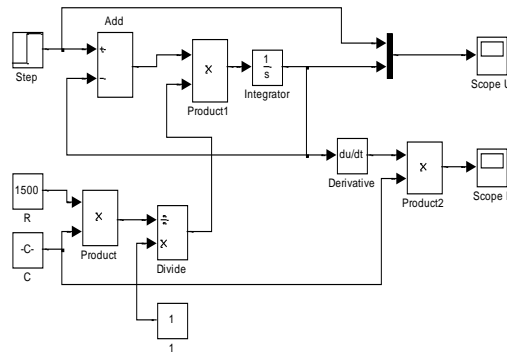


Fig.4. SIMULINK model of the RC circuit at closing

The k switcher is realized using the same voltage step signal which is applied at the terminal of the circuit, to simulate the closing of the k switcher.

The Simulink model from Fig.4 creates the diagrams from Fig.5 and Fig.6.

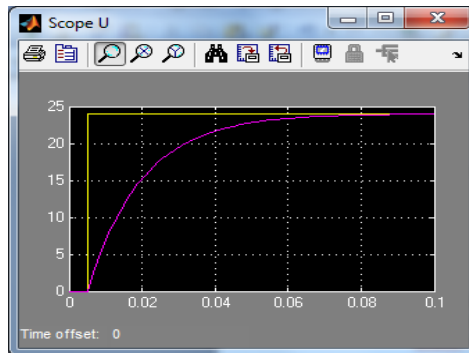


Fig.5. Capacitor voltage variation

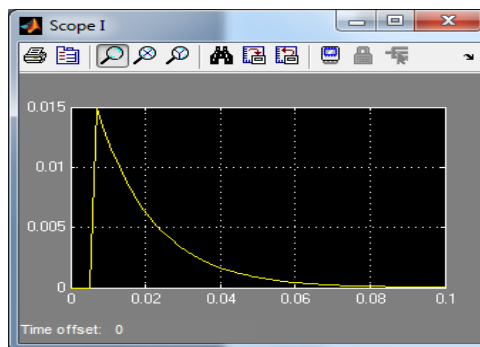


Fig.6. Current variation after closing

3. STUDY OF REACTIVE SERIES CIRCUITS

We will consider the RLC series circuit with concentrated parameters. At the initial moment, the k circuit switcher is closed and we intend to study the behavior and variation of circuit electrical parameters after connection [3], [5-6].

The integral-differential equation which corresponds to the transitory regime of the considered circuit is the following:

$$Ri + L \frac{di}{dt} + \frac{1}{C} \int idt = E \quad (11)$$

or:

$$LC \frac{d^2u}{dt^2} + RC \frac{du}{dt} + u = E \quad (12)$$

The following notations are made:

$$\delta = \frac{R}{2L} \quad (13)$$

The circuit amortization:

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (14)$$

The circuit personal pulsation.

$$\omega = \sqrt{\delta^2 - \omega_0^2} \quad (15)$$

We will consider the situation when $\delta > \omega_0$ or $R < 2\sqrt{L/C}$, which will be checked by the resistor in the circuit.

Consequently, the solving of the differential equation gives the following solutions:

$$u(t) = E \left[1 - \frac{1}{2\sqrt{\delta^2 - \omega_0^2}} (r_1 e^{r_1 t} - r_2 e^{r_2 t}) \right] \quad (16)$$

$$i(t) = \frac{E}{2L\sqrt{\delta^2 - \omega_0^2}} (e^{r_1 t} - e^{r_2 t}) \quad (17)$$

where r1 and r2 are the roots of the characteristic equation:

$$\begin{aligned} r_1 &= -\delta + \omega \\ r_2 &= -\delta - \omega \end{aligned} \quad (18)$$

We will consider the situation when $\delta < \omega_0$ or $R > 2\sqrt{L/C}$. The following notation was made:

$$\omega_0^2 - \delta^2 = \omega'^2 \quad (19)$$

Consequently, the solving of the differential equation gives the following solutions:

$$u(t) = E \left[1 - \frac{\omega_0}{\omega'} \cdot e^{-\delta t} \cdot \sin(\omega' t + \beta') \right] \quad (20)$$

$$i(t) = \frac{E}{\omega' L} e^{-\delta t} \sin \omega' t \quad (21)$$

where:
$$\beta' = \arccos \frac{\delta}{\omega_0} \quad (22)$$

3.1 SIMULINK model of the circuit

The SIMULINK model of the circuit after closing was done on grounds of the second order differential equation (12), which is put in the form (23) where the higher order derivative is separated:

$$\frac{d^2 u}{dt} = \frac{1}{LC} \left[E - RC \frac{du}{dt} - u \right] \quad (23)$$

The SIMULINK model from Fig.7 generates the voltage capacitor and the current through the circuit during the transitory regime. To this purpose, two values of the R resistor are considered, which correspond to two important regimes:

- Aperiodic regime;
- Oscillatory regime.

For each regime the capacitor voltage and the circuit current variation diagrams are plotted.

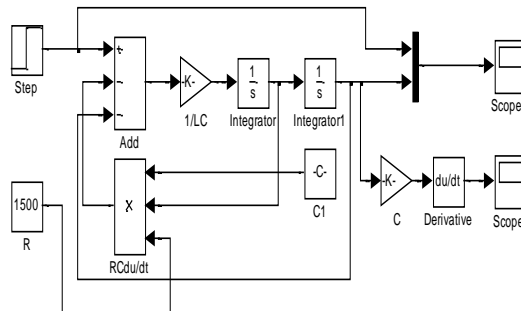


Fig.7. SIMULINK model of the circuit

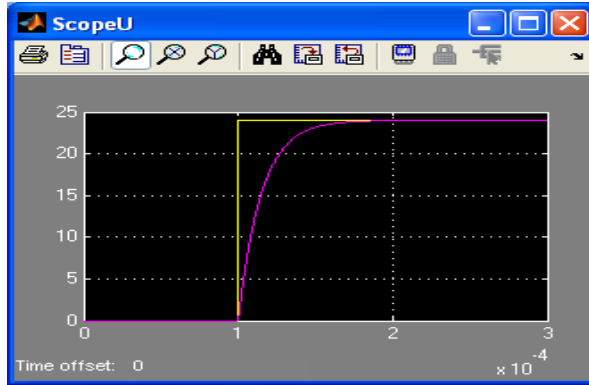


Fig.8. Voltage variation of capacitor in the a-periodic mode

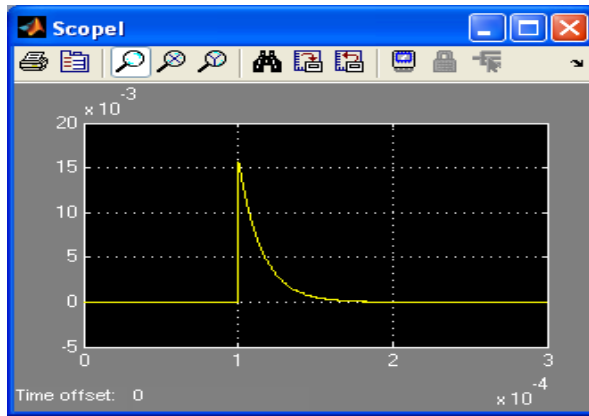


Fig.9. Current variation in the a- periodic mod

The SIMULINK model from Fig.7 leads to obtain the following MATLAB diagrams, Fig. 8, Fig.9 according to the equations (16) and (17) for a-periodic mode, and Fig.10, Fig.11 according to the equations (20) and (21) for oscillating mode [1].

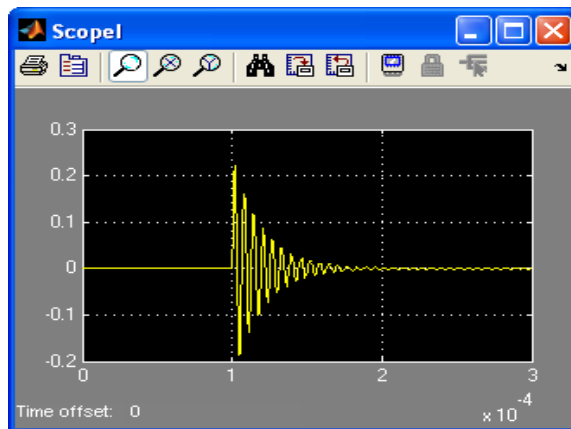


Fig.10. Voltage variation of capacitor in the oscillating regime

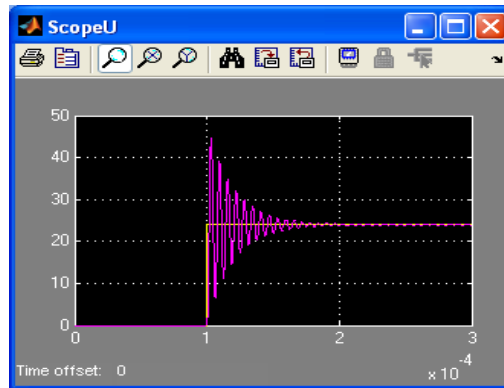


Fig.11. Current variation in the oscillating regime

The k switcher is realized by using a voltage step signal which is applied at the terminal of the circuit, to simulate the closing of the k switcher. Changing the value of resistance, automatically leads to the updating of the voltage and current charts

4. CONCLUSIONS

An important conclusion of this paper is that the study can be extended to some more complex circuits and these can be studied with this modern method which involves the use of virtual medium.

In order to realize the simulating model which integrates the differential equations, the basic idea is to separate in the left member of equation, the higher order derivative. The analytical expressions obtained in the right member of the equation underlie the achievement of the simulating model by means of specific blocks of the virtual medium. In the case of complex circuits, the simulating model is conceived on a differential equations system.

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